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# On the Coefficient Bounds of Bi-Univalent Functions Defined by Quasi-Subordination

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**Abstract**

In this paper, we introduce and examine two subclasses $\mathcal{M}_{q,\Sigma}^{\beta}(\varsigma,\mu,\varphi)$ and $\ell_{q,\Sigma}(\delta,\varepsilon,\varphi)$ of analytic bi-univalent functions in the open unit disk $\mathcal{U}$ by means of quasi-subordination. For functions belonging to these subclasses,we derive upper esimates for the initial Taylor–Maclaurin coefficients $|a_2|$ and $|a_3|$. Furthermore, certain special cases of the obtained results are discussed.

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## Introduction

Let H be the family of all normalized analytic functions thoughout the open unit disk $\mathcal{U} = \{\zeta: |\zeta| < 1\}$, of the form:

$$f(\zeta) = \zeta + \sum_{j=2}^{\infty} a_j \zeta^j, \qquad (\zeta \in \mathcal{U}). \tag{1.1}$$

Let M be the family of normalized analytic functions that are univalent thoughout the open unit disk $\mathcal{U}$.
For every $f \in$ M, the inverse function $f^{-1}$ is analytic in a disk centered at the origin with radius at least $1/4$ . Hence, as guaranteed by the Koebe One-Quarter Theorem:

$$f^{-1}(f(\zeta)) = \zeta, \qquad (\zeta \in \mathcal{U})$$

And

$$f(f^{-1}(\eta)) = \eta \qquad (|\eta| < r_0(f), r_0(f) \geq \tfrac{1}{4})$$

where

$$f^{-1}(\eta) = g(\eta) = \eta - a_2\eta^2 + (2a_2^2 - a_3)\eta^3 - (5a_2^3 - 5a_2a_3 + a_4)\eta^4 + \cdots. \tag{1.2}$$

The inverse function $f^{-1}$ is analytic only in a disk $|\eta| < r_0(f)$, as illustrated in Figure 1 for $f(\zeta) = \zeta/(1-\zeta)$.

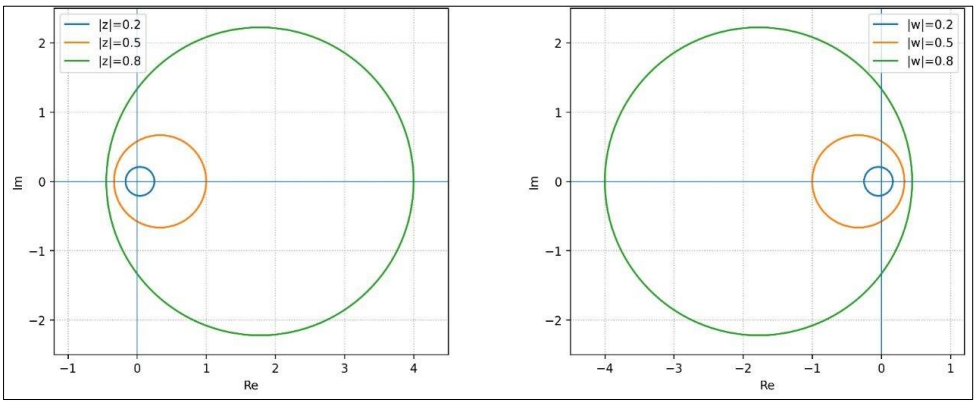



**Fig 1:** Images of circles under $f(\zeta) = \frac{\zeta}{1-\zeta}$ (left) and its inverse $f^{-1}(\eta) = \frac{\eta}{1+\eta}$ (right).

A function $f$ is called bi-univalent thoughout $\mathcal{U}$, provided that both $f$ and its inverse function are univalent. The family of bi-univalent functions in $\mathcal{U}$ is denoted by $\sum$.
Let $f$ and $\varphi$ be analytic functions thoughout $\mathcal{U}$. We say that $f$ is quasi-subordinate to $\varphi$, denoted by:

$$f \prec_q \varphi \ \ or \ \ f(\zeta) \prec_q \varphi(\zeta), \qquad (\zeta \in \mathcal{U}).$$

If there exist analytic functions $\psi$ and $\omega$ in $\mathcal{U}$ satisfying

$$|\psi(\zeta)| \leq 1, \ \omega(0) = 0 \text{ and } |\omega(\zeta)| < 1$$

such that

$$f(\zeta) = \psi(\zeta)\varphi(\omega(\zeta)), \ \ (\zeta \in \mathcal{U}).$$

In particular, when $\psi(\zeta) \equiv 1$, the quasi-subordination condition reduces to $f(\zeta) = \varphi(\omega(\zeta))$, where $\omega(0) = 0$ and $|\omega(\zeta)| < 1$ for $\zeta \in \mathcal{U}$. Consequently, $f(\zeta) \prec \varphi(\zeta)$ in $\mathbb{U}$, which represents the usual subordination. On the other hand, if $\omega(\zeta) = \zeta$, then the relation becomes $f(\zeta) = \psi(\zeta)\varphi(\zeta)$, where $|\psi(\zeta)| \leq 1$.
In this case, $f$ is said to be majorized by $\varphi$, and this relation is denoted by $f(\zeta) \ll \varphi(\zeta)$ in $\mathcal{U}$. Therefore, quasi-subordination generalizes both subordination and majorization.
Lewin [10] introduced the family of bi-univalent functions in 1967 and obtained the estimate $|a_2| \leq 1.51$. Subsequently, Brannan and Clunie [4] proposed the conjectured that $|a_2| \leq \sqrt{2}$. Later, Styer and Wright [17] demonstrated the existence of a function $f \in \Sigma$ for which $|a_2| > \frac{4}{3}$, whereas Tan [18] obtained the upper bound $|a_2| < 1.485$. [16] examined subclasses of analytic bi-univalent functions and derived bounds for the initial coefficients $|a_2|$ and $|a_3|$ [2].
Ma and Minda [11] introduced claasses of starlike and convex functions by means of subordination and the following classes:

$$S^*(\varphi) = \{f \in \mathrm{H}: \frac{\zeta f'(\zeta)}{f(\zeta)} \prec \varphi(\zeta), \ \zeta \in \mathcal{U}\},$$

and

$$G^*(\varphi) = \{f \in \mathrm{H}: 1 + \frac{\zeta f''(\zeta)}{f'(\zeta)} \prec \varphi(\zeta), \ \zeta \in \mathcal{U}\},$$

Throughout this work, we required that $\varphi$ has the following expansion:

$$\varphi(\zeta) = 1 + P_1\zeta + P_2\zeta^2 + P_3\zeta^3 + \cdots, \qquad \zeta \in \mathcal{U} \ \ (P_2 \geq P_1 > 0). \tag{1.3}$$

Consequently,

$$\varphi(0) = 1, \text{and } \varphi'(0) > 0$$

Moreover, the auxiliary function $\psi$ is assumed to be analytic and bounded in $\mathcal{U}$, and is given by

$$\psi(\zeta) = Y_0 + Y_1\zeta + Y_2\zeta^2 + \cdots, \qquad \zeta \in \mathcal{U}. \tag{1.4}$$

Several studies have investigated coefficient estimates for analytic bi-univalent functions, particularly upper bounds for the initial coefficients $|a_2|$ and $|a_3|$ [1, 7, 8, 9, 15, 19, 20].

Moreover, other studies have focused on the introducing and investigating subclasses of analytic bi-univalent functions [3, 5, 12, 13, 14].

**Lemma (1.1)** [6]: Let $s(\zeta) = 1 + s_1\zeta + s_2\zeta^2 + s_3\zeta^3 + \cdots \in \mathcal{P}$, where $\mathcal{P}$ denotes the class of functions that are analytic in $\mathcal{U}$, and satisfy

$$Re\{s(\zeta)\} > 0, \quad (\zeta \in \mathcal{U}).$$

Then

$$|s_n| \leq 2 \ (n = 1, 2, 3, \ldots).$$

**Main Results**

**The Subclass $\mathcal{M}_{q,\Sigma}^{\beta}(\lambda, \mu, \vartheta)$**

**Definition (2.1):** A function $f \in$ H defined by (1.1) is considered to belong to the class $\mathcal{M}_{q,\Sigma}^{\beta}(\varsigma, \mu, \varphi)$, provided that it fulfills the following quasi-subordination conditions:

$$\frac{1}{\beta}\left[\left(\frac{\zeta f'(\zeta)}{f(\zeta)}\right)\left(1 + \varsigma\frac{\zeta f''(\zeta)}{f'(\zeta)} + \mu\frac{\zeta^2 f'''(\zeta)}{f'(\zeta)}\right) - 1\right] \prec_q (\varphi(\zeta) - 1), \tag{2.1}$$

and

$$\frac{1}{\beta}\left[\left(\frac{\eta g'(\eta)}{g(\eta)}\right)\left(1 + \varsigma\frac{\eta g''(\eta)}{g'(\eta)} + \mu\frac{\eta^2 g'''(\eta)}{g'(\eta)}\right) - 1\right] \prec_q (\varphi(\eta) - 1), \tag{2.2}$$

where ( $0 \leq \varsigma \leq 1, 0 \leq \mu \leq 1$ and $\beta \in \mathbb{C}\backslash\{0\}$), and $g = f^{-1}$ and $\varphi$ as defined in (1.3) and $\zeta, \eta \in \mathcal{U}$.

**Remark(2.1):** If $\mu = 0$ in Definition (2.1), then the class $\mathcal{M}_{q,\Sigma}^{\beta}(\varsigma, 0, \varphi)$ formed by functions $f \in$ H, given by (1.1), subject to the quasi-subordination conditions stated below:

$$\frac{1}{\beta}\left[\left(\frac{\zeta f'(\zeta)}{f(\zeta)}\right)\left(1 + \varsigma\frac{\zeta f''(\zeta)}{f'(\zeta)}\right) - 1\right] \prec_q (\varphi(\zeta) - 1)$$

and

$$\frac{1}{\beta}\left[\left(\frac{\eta g'(\eta)}{g(\eta)}\right)\left(1 + \varsigma\frac{\eta g''(\eta)}{g'(\eta)}\right) - 1\right] \prec_q (\varphi(\eta) - 1).$$

**Remark(2.2):** If $\varsigma = 0$ in Definition (2.1), then the class $\mathcal{M}_{q,\Sigma}^{\beta}(0, \mu, \varphi)$ formed by functions $f \in$ H, given by (1.1), subject to the quasi-subordination conditions stated below:

$$\frac{1}{\beta}\left[\left(\frac{\zeta f'(\zeta)}{f(\zeta)}\right)\left(1 + \mu\frac{\zeta^2 f'''(\zeta)}{f'(\zeta)}\right) - 1\right] \prec_q (\varphi(\zeta) - 1),$$

and

$$\frac{1}{\beta}\left[\left(\frac{\eta g'(\eta)}{g(\eta)}\right)\left(1 + \mu\frac{\eta^2 g'''(\eta)}{g'(\eta)}\right) - 1\right] \prec_q (\varphi(\eta) - 1).$$

**Remark(2.3):** If $\mu = 0$ and $\varsigma = 0$ in Definition (2.1), then the class $\mathcal{M}_{q,\Sigma}^{\beta}(0, 0, \varphi)$, formed by functions $f \in$ H, given by (1.1), subject to the quasi-subordination conditions stated below:

$$\frac{1}{\beta}\left[\left(\frac{\zeta f'(\zeta)}{f(\zeta)}\right) - 1\right] \prec_q (\varphi(\zeta) - 1),$$

and

$$\frac{1}{\beta}\left[\left(\frac{\eta g'(\eta)}{g(\eta)}\right) - 1\right] \prec_q (\varphi(\eta) - 1).$$

**Remark(2.4):** If $\mu = 0$ and $\varsigma = 1$ in Definition (2.1), then the class $\mathcal{M}_{q,\Sigma}^{\beta}(1, 0, \varphi)$, formed by functions $f \in$ H, given by (1.1), subject to the quasi-subordination conditions stated below:

$$\frac{1}{\beta}\left[\left(\frac{\zeta f'(\zeta)}{f(\zeta)}\right)\left(1 + \frac{\zeta f''(\zeta)}{f'(\zeta)}\right) - 1\right] \prec_q (\varphi(\zeta) - 1),$$

and

$$\frac{1}{\beta}\left[\left(\frac{\eta g'(\eta)}{g(\eta)}\right)\left(1 + \frac{\eta g''(\eta)}{g'(\eta)}\right) - 1\right] \prec_q (\varphi(\eta) - 1).$$

**Theorem (2.1):** Let $0 \leq \varsigma \leq 1, 0 \leq \mu \leq 1$ and $\beta \in \mathbb{C}\backslash\{0\}$. If $f(\zeta)$ belongs to the class $\mathcal{M}_{q,\Sigma}^{\beta}(\varsigma, \mu, \varphi)$, then

$$|a_2| \leq min\{\frac{|\beta\gamma_0|P_1}{(1+2\varsigma)}, \sqrt{\frac{|\beta\gamma_0|P_2}{(1+4\varsigma+6\mu)}}\}, \tag{2.3}$$

and

$$|a_3| \leq min\{\frac{|\beta\gamma_0|P_2}{(1+4\varsigma+6\mu)} + \frac{|\beta|P_1(|\gamma_0|+|\gamma_1|)}{2(1+3\varsigma+3\mu)}, \frac{|\beta|P_1(|\gamma_0|+|\gamma_1|)}{2(1+3\varsigma+3\mu)} + \frac{|\beta\gamma_0|^2P_1^2}{(1+2\varsigma)^2}\}. \tag{2.4}$$

**Proof:** Assume that $f$ belongs to $\mathcal{M}_{q,\Sigma}^{\beta}(\varsigma, \mu, \varphi)$. Accordingly, there exist two analytic functions $\mathrm{d}, \mathrm{t}: \mathcal{U} \to \mathcal{U}$, with $\mathrm{d}(0) = \mathrm{t}(0) = 0$, $|\mathrm{d}(\zeta)| < 1$ and $|\mathrm{t}(\eta)| < 1$. Then the function $\psi$ defined by $(1.4)$, satisfies:

$$\frac{1}{\beta}[(\frac{\zeta f'(\zeta)}{f(\zeta)})(1 + \varsigma\frac{\zeta f''(\zeta)}{f'(\zeta)} + \mu\frac{\zeta^2 f'''(\zeta)}{f'(\zeta)}) - 1] = \psi(\zeta)(\varphi(\mathrm{d}(\zeta)) - 1), \tag{2.5}$$

and

$$\frac{1}{\beta}[(\frac{\eta g'(\eta)}{g(\eta)})(1 + \varsigma\frac{\eta g''(\eta)}{g'(\eta)} + \mu\frac{\eta^2 g'''(\eta)}{g'(\eta)}) - 1] = \psi(\eta)(\varphi(\mathrm{t}(\eta)) - 1). \tag{2.6}$$

Define the functions $\mathrm{V}(\zeta)$ and $\mathrm{W}(\eta)$ by

$$\mathrm{V}(\zeta) = \frac{1+\mathrm{d}(\zeta)}{1-\mathrm{d}(\zeta)} = 1 + \mathrm{h}_1\zeta + \mathrm{h}_2\zeta^2 + \cdots, \tag{2.7}$$

and

$$\mathrm{W}(\eta) = \frac{1+\mathrm{t}(\eta)}{1-\mathrm{t}(\eta)} = 1 + \mathrm{k}_1\eta + \mathrm{k}_2\eta^2 + \cdots. \tag{2.8}$$

Or equivalently

$$\mathrm{d}(\zeta) = \frac{\mathrm{V}(\zeta)-1}{\mathrm{V}(\zeta)+1} = \frac{1}{2}[\mathrm{h}_1\zeta + (\mathrm{h}_2 - \frac{\mathrm{h}_1^2}{2})\zeta^2 + \cdots], \tag{2.9}$$

and

$$\mathrm{t}(\eta) = \frac{\mathrm{W}(\eta)-1}{\mathrm{W}(\eta)+1} = \frac{1}{2}[\mathrm{k}_1\eta + (\mathrm{k}_2 - \frac{\mathrm{k}_1^2}{2})\eta^2 + \cdots]. \tag{2.10}$$

Since $\mathrm{d}, \mathrm{t}: \mathcal{U} \to \mathcal{U}$, the functions $\mathrm{V}(\zeta)$ and $\mathrm{W}(\eta)$ are analytic throughout $\mathcal{U}$ and fulfill $Re\{\mathrm{V}(\zeta)\} > 0$ and $Re\{\mathrm{W}(\zeta)\} > 0$. Furthemore $\mathrm{V}(0) = \mathrm{W}(0) = 1$.
By substituting the expressions given in $(2.9)$ and $(2.10)$ into equations $(2.5)$ and $(2.6)$, respectively, we obtain:

$$\frac{1}{\beta}[(\frac{\zeta f'(\zeta)}{f(\zeta)})(1 + \varsigma\frac{\zeta f''(\zeta)}{f'(\zeta)} + \mu\frac{\zeta^2 f'''(\zeta)}{f'(\zeta)}) - 1] = \psi(\zeta)(\varphi(\frac{\mathrm{V}(\zeta)-1}{\mathrm{V}(\zeta)+1}) - 1), \tag{2.11}$$

and

$$\frac{1}{\beta}[(\frac{\eta g'(\eta)}{g(\eta)})(1 + \varsigma\frac{\eta g''(\eta)}{g'(\eta)} + \mu\frac{\eta^2 g'''(\eta)}{g'(\eta)}) - 1] = \psi(\eta)(\varphi(\frac{\mathrm{W}(\eta)-1}{\mathrm{W}(\eta)+1}) - 1). \tag{2.12}$$

Substituting the Taylor expansions of f(z) and g(w), as given in $(1.1)$ and $(1.2)$, respectively, into the above expressions, we obtain:

$$\frac{1}{\beta}[(\frac{\zeta f'(\zeta)}{f(\zeta)})(1 + \lambda\frac{\zeta f''(\zeta)}{f'(\zeta)} + \mu\frac{\zeta^2 f'''(\zeta)}{f'(\zeta)}) - 1] = \frac{1}{\beta}[(1 + 2\varsigma)a_2\zeta + (2(1 + 3\varsigma + 3\mu)a_3 - (1 + 2\varsigma)a_2^2)\zeta^2 + \cdots] \tag{2.13}$$

$$\frac{1}{\beta}[(\frac{\eta g'(\eta)}{g(\eta)})(1 + \varsigma\frac{\eta g''(\eta)}{g'(\eta)} + \mu\frac{\eta^2 g'''(\eta)}{g'(\eta)}) - 1] == \frac{1}{\beta}[-(1 + 2\varsigma)a_2\eta + (2(1 + 3\varsigma + 3\mu)(2a_2^2 - a_3) - (1 + 2\varsigma)a_2^2)\eta^2 + \cdots] \tag{2.14}$$

Combining the expressions in $(2.9)$ and $(2.10)$ with the expansions given in $(1.3)$ and $(1.4)$, we obtain:

$$\psi(\zeta)(\varphi(\frac{\mathrm{V}(\zeta)-1}{\mathrm{V}(\zeta)+1}) - 1) = \frac{1}{2}\gamma_0 P_1\mathrm{h}_1\zeta + (\frac{1}{2}\gamma_1 P_1\mathrm{h}_1 + \frac{1}{2}\gamma_0 P_1(\mathrm{h}_2 - \frac{\mathrm{h}_1^2}{2}) + \frac{1}{4}\gamma_0 P_2\mathrm{h}_1^2)\zeta^2 + \cdots, \tag{2.15}$$

and

$$\psi(\eta)(\varphi(\frac{\mathrm{W}(\eta)-1}{\mathrm{W}(\eta)+1}) - 1) = \frac{1}{2}\gamma_0 P_1\mathrm{k}_1\eta + (\frac{1}{2}\gamma_1 P_1\mathrm{k}_1 + \frac{1}{2}\gamma_0 P_1(\mathrm{k}_2 - \frac{\mathrm{k}_1^2}{2}) + \frac{1}{4}\gamma_0 P_2\mathrm{k}_1^2)\eta^2 + \cdots. \tag{2.16}$$

By equating the matching coefficients in $(2.13)$ and $(2.15)$, yields the following relations:

$$\frac{1}{\beta}(1+2\varsigma)a_2 = \frac{1}{2}\Upsilon_0 P_1 \mathrm{h}_1, \tag{2.17}$$

and

$$\frac{1}{\beta}[2(1+3\varsigma+3\mu)a_3 - (1+2\varsigma)a_2^2] = \frac{1}{2}\Upsilon_1 P_1 \mathrm{h}_1 + \frac{1}{2}\Upsilon_0 P_1(\mathrm{h}_2 - \frac{\mathrm{h}_1^2}{2}) + \frac{1}{4}\Upsilon_0 P_2 \mathrm{h}_1^2. \tag{2.18}$$

Similarly, comparison of the corresponding coefficients in $(2.14)$ and $(2.16)$ gives:

$$-\frac{1}{\beta}(1+2\varsigma)a_2 = \frac{1}{2}\Upsilon_0 P_1 \mathrm{k}_1, \tag{2.19}$$

and

$$\frac{1}{\beta}[2(1+3\varsigma+3\mu)(2a_2^2 - a_3) - (1+2\varsigma)a_2^2] = \frac{1}{2}\Upsilon_1 P_1 \mathrm{k}_1 + \frac{1}{2}\Upsilon_0 P_1(\mathrm{k}_2 - \frac{\mathrm{k}_1^2}{2}) + \frac{1}{4}\Upsilon_0 P_2 \mathrm{k}_1^2. \tag{2.20}$$

Combining the relations $(2.17)$ and $(2.19)$, we obtain:

$$a_2 = \frac{\beta\Upsilon_0 P_1 \mathrm{h}_1}{2(1+2\varsigma)} = -\frac{\beta\Upsilon_0 P_1 \mathrm{k}_1}{2(1+2\varsigma)}. \tag{2.21}$$

Consequently

$$\mathrm{h}_1 = -\mathrm{k}_1, \tag{2.22}$$

and

$$8(1+2\varsigma)^2 a_2^2 = \beta^2 \Upsilon_0^2 P_1^2(\mathrm{h}_1^2 + \mathrm{k}_1^2). \tag{2.23}$$

Applying Lemma $(1.1)$ to equation $(2.23)$, we obtain:

$$|a_2| \leq \frac{|\beta\Upsilon_0| P_1}{(1+2\varsigma)}.$$

Adding equations $(2.18)$ and $(2.20)$ yields the following relation:

$$4[4(1+3\varsigma+3\mu) - 2(1+2\varsigma)]a_2^2 = 2\beta\Upsilon_0 P_1(\mathrm{h}_2 + \mathrm{k}_2) + \beta\Upsilon_0(\mathrm{h}_1^2 + \mathrm{k}_1^2)(P_2 - P_1). \tag{2.24}$$

Consequently,

$$a_2^2 = \frac{\beta\Upsilon_0[2P_1(\mathrm{h}_2+\mathrm{k}_2)+(\mathrm{h}_1^2+\mathrm{k}_1^2)(P_2-P_1)]}{8(1+4\varsigma+6\mu)}. \tag{2.25}$$

An application of Lemma $(1.1)$ to equation $(2.25)$ yields:

$$|a_2| \leq \sqrt{\frac{|\beta\Upsilon_0| P_2}{(1+4\varsigma+6\mu)}}.$$

To derive an estimate for $|a_3|$, we subtract equation $(2.20)$ from equation $(2.18)$, which gives:

$$\frac{4}{\beta}[4(1+3\varsigma+3\mu)a_3 - 4(1+3\varsigma+3\mu)a_2^2] = 2\Upsilon_1 P_1(\mathrm{h}_1 - \mathrm{k}_1) + 2\Upsilon_0 P_1(\mathrm{h}_2 - \mathrm{k}_2) + \Upsilon_0(P_2 - P_1)(\mathrm{h}_1^2 - \mathrm{k}_1^2). \tag{2.26}$$

Combining the relation $(2.26)$ with the estimate obtained in $(2.25)$, we arrive at:

$$|a_3| \leq \frac{|\beta\Upsilon_0| P_2}{(1+4\varsigma+6\mu)} + \frac{|\beta| P_1(|\Upsilon_0|+|\Upsilon_1|)}{2(1+3\varsigma+3\mu)}.$$

Furthermore, another estimate for $|a_3|$ is obtained as follows:

$$|a_3| \leq \frac{|\beta| P_1(|\Upsilon_0|+|\Upsilon_1|)}{2(1+3\varsigma+3\mu)} + \frac{|\beta\Upsilon_0|^2 P_1^2}{(1+2\varsigma)^2}.$$

The proof is complete. ■
Setting $\mu = 0$ in Theorem $(2.1)$, leads to the result below.
**Corollary (2.1):** Let $f \in H$ belong to $\mathcal{M}_{q,\Sigma}^{\beta}(\varsigma, 0, \varphi)$. Then

$$|a_2| \leq min\{\frac{|\beta\Upsilon_0|P_1}{(1+2\varsigma)}, \sqrt{\frac{|\beta\Upsilon_0|P_2}{(1+4\varsigma)}}\},$$

and

$$|a_3| \leq min\{\frac{|\beta\Upsilon_0|P_2}{(1+4\varsigma)} + \frac{|\beta|P_1(|\Upsilon_0|+|\Upsilon_1|)}{2(1+3\varsigma)}, \frac{|\beta|P_1(|\Upsilon_0|+|\Upsilon_1|)}{2(1+3\varsigma)} + \frac{|\beta\Upsilon_0|^2P_1^2}{(1+2\varsigma)^2}\}.$$

putting $\varsigma = 0$ in Theorem $(2.1)$, leads to the result below.
**Corollary (2.2):** Let $f \in H$ belong to $\mathcal{M}_{q,\Sigma}^{\beta}(0, \mu, \varphi)$. Then

$$|a_2| \leq min\{|\beta\Upsilon_0|P_1, \sqrt{\frac{|\beta\Upsilon_0|P_2}{(1+6\mu)}}\},$$

and

$$|a_3| \leq min\{\frac{|\beta\Upsilon_0|P_2}{(1+6\mu)} + \frac{|\beta|P_1(|\Upsilon_0|+|\Upsilon_1|)}{2(1+3\mu)}, \frac{|\beta|P_1(|\Upsilon_0|+|\Upsilon_1|)}{2(1+3\mu)} + |\beta\Upsilon_0|^2P_1^2\}.$$

Putting $\mu = 0$ and $\varsigma = 0$ in Theorem $(2.1)$, leads to the result below.
**Corollary(2.3):** Let $f \in H$ belong to $\mathcal{M}_{q,\Sigma}^{\beta}(\varsigma, \mu, \varphi)$. Then

$$|a_2| \leq min\{|\beta\Upsilon_0|P_1, \sqrt{|\beta\Upsilon_0|P_2}\},$$

and

$$|a_3| \leq min\{|\beta\Upsilon_0|P_2 + \frac{1}{2}|\beta|P_1(|\Upsilon_0| + |\Upsilon_1|), \frac{1}{2}|\beta|P_1(|\Upsilon_0| + |\Upsilon_1|) + |\beta\Upsilon_0|^2P_1^2\}.$$

Putting $\mu = 0$ and $\varsigma = 1$ in Theorem $(2.1)$, leads to the result below.
**Corollary (2.4):** Let $f \in H$ belong to $\mathcal{M}_{q,\Sigma}^{\beta}(\varsigma, \mu, \varphi)$. Then

$$|a_2| \leq min\{\frac{1}{3}|\beta\Upsilon_0|P_1, \sqrt{\frac{1}{5}|\beta\Upsilon_0|P_2}\},$$

and

$$|a_3| \leq min\{\frac{1}{5}|\beta\Upsilon_0|P_2 + \frac{1}{8}|\beta|P_1(|\Upsilon_0| + |\Upsilon_1|), \frac{1}{8}|\beta|P_1(|\Upsilon_0| + |\Upsilon_1|) + \frac{1}{9}|\beta\Upsilon_0|^2P_1^2\}.$$

## 3. The Subclass $\ell_{q,\Sigma}(\delta, \varepsilon, \vartheta)$

**Definition (3.1):** A function $f \in H$ given by $(1.1)$ is considered to belong to the class $\ell_{q,\Sigma}(\delta, \varepsilon, \varphi)$, provided that it fulfills the following quasi-subordination conditions:

$$\frac{\zeta f'(\zeta)}{f(\zeta)}[1 + \frac{\zeta^2 f'\prime(\zeta)}{f(\zeta)} + \delta\zeta(\frac{\zeta f'(\zeta)}{f(\zeta)})']^{\varepsilon} - 1 \prec_q (\varphi(\zeta) - 1), \quad (3.1)$$

and

$$\frac{\eta g'(\eta)}{g(\eta)}[1 + \frac{\eta^2 g'\prime(\eta)}{g(\eta)} + \delta\eta(\frac{\eta g'(\eta)}{g(\eta)})']^{\varepsilon} - 1 \prec_q (\varphi(\eta) - 1). \quad (3.2)$$

Where ( $\varepsilon \geq 1$, and $\delta \geq 0$), and $g = f^{-1}$ given by $(1.2)$, $\varphi$ given by $(1.3)$ and $\zeta, \eta \in \mathcal{U}$.
**Remark$(3.1)$:** If $\delta = 0$ in Definition $(3.1)$, then the class $\ell_{q,\Sigma}(0, \varepsilon, \varphi)$, formed by functions $f \in H$, given by $(1.1)$, subject to the quasi-subordination conditions stated below:

$$\frac{\zeta f'(\zeta)}{f(\zeta)}[1 + \frac{\zeta^2 f'\prime(\zeta)}{f(\zeta)}]^{\varepsilon} - 1 \prec_q (\varphi(\zeta) - 1),$$

and

$$\frac{\eta g'(\eta)}{g(\eta)}[1 + \frac{\eta^2 g'\prime(\eta)}{g(\eta)}]^{\varepsilon} - 1 \prec_q (\varphi(\eta) - 1).$$

**Remark**$(3.2)$ **:**If $\varepsilon = 1$ and $\delta = 0$ in Definition $(3.1)$, then the class $\ell_{q,\Sigma}(0,1,\varphi)$, formed by functions $f \in \mathrm{H}$, given by $(1.1)$, subject to the quasi-subordination conditions stated below:

$$\frac{\zeta f'(\zeta)}{f(\zeta)}\left[1+\frac{\zeta^2 f'\prime(\zeta)}{f(\zeta)}\right]-1 \prec_q (\varphi(\zeta)-1),$$

and

$$\frac{\eta g'(\eta)}{g(\eta)}\left[1+\frac{\eta^2 g'\prime(\eta)}{g(\eta)}\right]-1 \prec_q (\varphi(\eta)-1).$$

**Theorem (3.1):** Let $\varepsilon \geq 1$, and $\delta \geq 0$. If $f(\zeta)$ belongs to the class $\ell_{q,\Sigma}(\delta,\varepsilon,\varphi)$. Then:

$$|a_2| \leq min\{\frac{|\Upsilon_0|P_1}{(1+\varepsilon(2+\delta))}, \sqrt{\frac{|\Upsilon_0|P_2}{1+6\varepsilon+3\varepsilon\delta+\frac{\varepsilon(\varepsilon-1)}{2}(2+\delta)^2}}\}, \tag{3.3}$$

and

$$|a_3| \leq min\{\frac{|\Upsilon_0|P_2}{(1+6\varepsilon+3\varepsilon\delta+\frac{\varepsilon(\varepsilon-1)}{2}(2+\delta)^2)}+\frac{P_1(|\Upsilon_0|+|\Upsilon_1|)}{(2+6\varepsilon+4\varepsilon\delta)}, \frac{P_1(|\Upsilon_0|+|\Upsilon_1|)}{(2+6\varepsilon+4\varepsilon\delta)}+\frac{|\Upsilon_0|^2P_1^2}{(1+\varepsilon(2+\delta))^2}\}. \tag{3.4}$$

**Proof:** Assume that $f$ belongs to $\ell_{q,\Sigma}(\delta,\varepsilon,\varphi)$. Accordingly, there exist two analytic functions $\mathrm{d}_\wp, \mathrm{t}_\wp: \mathcal{U} \to \mathcal{U}$, with $\mathrm{d}_\wp(0) = \mathrm{t}_\wp(0) = 0$ , $|\mathrm{d}_\wp(\zeta)| < 1$ and $|\mathrm{t}_\wp(\eta)| < 1$. Then the function $\psi$ defined by $(1.4)$ satisfies:

$$\frac{\zeta f'(\zeta)}{f(\zeta)}\left[1+\frac{\zeta^2 f'\prime(\zeta)}{f(\zeta)}+\delta\zeta\left(\frac{\zeta f'(\zeta)}{f(\zeta)}\right)'\right]^{\varepsilon}-1 = \psi(\zeta)(\varphi(\mathrm{d}_\wp(\zeta))-1), \tag{3.5}$$

and

$$\frac{\eta g'(\eta)}{g(\eta)}\left[1+\frac{\eta^2 g'\prime(\eta)}{g(\eta)}+\delta\eta\left(\frac{\eta g'(\eta)}{g(\eta)}\right)'\right]^{\varepsilon}-1 = \psi(\eta)(\varphi(\mathrm{t}_\wp(\eta))-1). \tag{3.6}$$

Define the functions $\mathrm{V}(\zeta)$ and $\mathrm{W}(\eta)$ by

$$\mathrm{V}(\zeta) = \frac{1+\mathrm{d}_\wp(\zeta)}{1-\mathrm{d}_\wp(\zeta)} = 1+\mathrm{h}_1\zeta+\mathrm{h}_2\zeta^2+\cdots, \tag{3.7}$$

and

$$\mathrm{W}(\eta) = \frac{1+\mathrm{t}_\wp(\eta)}{1-\mathrm{t}_\wp(\eta)} = 1+\mathrm{k}_1\eta+\mathrm{k}_2\eta^2+\cdots. \tag{3.8}$$

Or equivalently

$$\mathrm{d}_\wp(\zeta) = \frac{\mathrm{V}(\zeta)-1}{\mathrm{V}(\zeta)+1} = \frac{1}{2}\left[\mathrm{h}_1\zeta+(\mathrm{h}_2-\frac{\mathrm{h}_1^2}{2})\zeta^2+\cdots\right], \tag{3.9}$$

and

$$\mathrm{t}_\wp(\eta) = \frac{\mathrm{W}(\eta)-1}{\mathrm{W}(\eta)+1} = \frac{1}{2}\left[\mathrm{k}_1\eta+(\mathrm{k}_2-\frac{\mathrm{k}_1^2}{2})\eta^2+\cdots\right]. \tag{3.10}$$

Since $\mathrm{d}_\wp, \mathrm{t}_\wp: \mathcal{U} \to \mathcal{U}$, the functions $\mathrm{V}(\zeta)$ and $\mathrm{W}(\eta)$ are analytic throughout $\mathcal{U}$ and fulfill $Re\{\mathrm{V}(\zeta)\} > 0$ and $Re\{\mathrm{W}(\zeta)\} > 0$. Furthemore $\mathrm{V}(0) = \mathrm{W}(0) = 1$.
By substituting the expressions given in $(3.9)$ and $(3.10)$ into equations $(3.5)$ and $(3.6)$, respectively, we obtain:

$$\frac{\zeta f'(\zeta)}{f(\zeta)}\left[1+\frac{\zeta^2 f'\prime(\zeta)}{f(\zeta)}+\delta\zeta\left(\frac{\zeta f'(\zeta)}{f(\zeta)}\right)'\right]^{\varepsilon}-1 = \psi(\zeta)(\varphi(\frac{\mathrm{V}(\zeta)-1}{\mathrm{V}(\zeta)+1})-1), \tag{3.11}$$

and

$$\frac{\eta g'(\eta)}{g(\eta)}\left[1+\frac{\eta^2 g'\prime(\eta)}{g(\eta)}+\delta\eta\left(\frac{\eta g'(\eta)}{g(\eta)}\right)'\right]^{\varepsilon}-1 = \psi(\eta)(\varphi(\frac{\mathrm{W}(\eta)-1}{\mathrm{W}(\eta)+1})-1). \tag{3.12}$$

Substituting the Taylor expansions of $f(\zeta)$ and $g(\eta)$, as given in $(1.1)$ and $(1.2)$, respectively, into the above expressions, we obtain:

$$\frac{\zeta f'(\zeta)}{f(\zeta)}[1+\frac{\zeta^2 f'\prime(\zeta)}{f(\zeta)}+\delta\zeta(\frac{\zeta f'(\zeta)}{f(\zeta)})']^{\varepsilon}-1=[(1+\varepsilon(2+\delta))a_2\zeta+((2+6\varepsilon+4\varepsilon\delta)a_3+(-1-\varepsilon\delta+\frac{\varepsilon(\varepsilon-1)}{2}(2+\delta)^2)a_2^2)\zeta^2+\cdots], \quad (3.13)$$

and

$$\frac{\eta g'(\eta)}{g(\eta)}[1+\frac{\eta^2 g'\prime(\eta)}{g(\eta)}+\delta\eta(\frac{\eta g'(\eta)}{g(\eta)})']^{\varepsilon}-1=[-(1+\varepsilon(2+\delta))a_2\eta+((2+6\varepsilon+4\varepsilon\delta)(2a_2^2-a_3)+(-1-\varepsilon\delta+\frac{\varepsilon(\varepsilon-1)}{2}(2+\delta)^2)a_2^2)\eta^2+\cdots]. \quad (3.14)$$

Combining the expressions in $(3.9)$ and $(3.10)$ with the expansions given in $(1.3)$ and $(1.4)$, we obtain:

$$\psi(\zeta)(\varphi(\frac{V(\zeta)-1}{V(\zeta)+1})-1)=\frac{1}{2}\Upsilon_0 P_1 h_1\zeta+(\frac{1}{2}\Upsilon_1 P_1 h_1+\frac{1}{2}\Upsilon_0 P_1(h_2-\frac{h_1^2}{2})+\frac{1}{4}\Upsilon_0 P_2 h_1^2)\zeta^2+\cdots, \quad (3.15)$$

and

$$\psi(\eta)(\varphi(\frac{W(\eta)-1}{W(\eta)+1})-1)=\frac{1}{2}\Upsilon_0 P_1 k_1\eta+(\frac{1}{2}\Upsilon_1 P_1 k_1+\frac{1}{2}\Upsilon_0 P_1(k_2-\frac{k_1^2}{2})+\frac{1}{4}\Upsilon_0 P_2 k_1^2)\eta^2+\cdots. \quad (3.16)$$

By equating the matching coefficients in $(3.13)$ and $(3.15)$, yields the following relations:

$$(1+\varepsilon(2+\delta))a_2=\frac{1}{2}\Upsilon_0 P_1 h_1, \quad (3.17)$$

and

$$[(2+6\varepsilon+4\varepsilon\delta)a_3+(-1-\varepsilon\delta+\frac{\varepsilon(\varepsilon-1)}{2}(2+\delta)^2)a_2^2]=\frac{1}{2}\Upsilon_1 P_1 h_1+\frac{1}{2}\Upsilon_0 P_1(h_2-\frac{h_1^2}{2})+\frac{1}{4}\Upsilon_0 P_2 h_1^2. \quad (3.18)$$

Similarly, comparison of the corresponding coefficients in $(3.14)$ and $(3.16)$ gives:

$$-(1+\varepsilon(2+\delta))a_2=\frac{1}{2}\Upsilon_0 P_1 k_1, \quad (3.19)$$

and

$$[(2+6\varepsilon+4\varepsilon\delta)(2a_2^2-a_3)+(-1-\varepsilon\delta+\frac{\varepsilon(\varepsilon-1)}{2}(2+\delta)^2)a_2^2]=\frac{1}{2}\Upsilon_1 P_1 k_1+\frac{1}{2}\Upsilon_0 P_1(k_2-\frac{k_1^2}{2})+\frac{1}{4}\Upsilon_0 P_2 k_1^2. \quad (3.20)$$

Combining the relations $(3.17)$ and $(3.19)$, we obtain:

$$a_2=\frac{\Upsilon_0 P_1 h_1}{2(1+\varepsilon(2+\delta))}=-\frac{\Upsilon_0 P_1 k_1}{2(1+\varepsilon(2+\delta))}. \quad (3.21)$$

Consequently

$$h_1=-k_1, \quad (3.22)$$

and

$$8(1+\varepsilon(2+\delta))^2 a_2^2=\Upsilon_0^2 P_1^2(h_1^2+k_1^2). \quad (3.23)$$

Applying Lemma $(1.1)$ to equation $(3.23)$, we obtain:

$$|a_2|\leq\frac{|\Upsilon_0|P_1}{(1+\varepsilon(2+\delta))}.$$

Adding equations $(3.18)$ and $(3.20)$ yields the following relation:

$$4[2(2+6\varepsilon+4\varepsilon\delta)+2(-1-\varepsilon\delta+\frac{\varepsilon(\varepsilon-1)}{2}(2+\delta)^2)]a_2^2=2\Upsilon_0 P_1(h_2+k_2)+\Upsilon_0(h_1^2+k_1^2)(P_2-P_1). \quad (3.24)$$

Consequently,

$$a_2^2=\frac{\Upsilon_0[2P_1(h_2+k_2)+(h_1^2+k_1^2)(P_2-P_1)]}{8(1+6\varepsilon+3\varepsilon\delta+\frac{\varepsilon(\varepsilon-1)}{2}(2+\delta)^2)}. \quad (3.25)$$

An application of Lemma $(1.1)$ to equation $(3.25)$ yields:

$$|a_2| \leq \sqrt{\frac{|\Upsilon_0|P_2}{1+6\varepsilon+3\varepsilon\delta+\frac{\varepsilon(\varepsilon-1)}{2}(2+\delta)^2}}.$$

To derive an estimate for $|a_3|$, we subtract equation $(3.20)$ from equation $(3.18)$, which gives:

$$4[2(2+6\varepsilon+4\varepsilon\delta)a_3 - 2(2+6\varepsilon+4\varepsilon\delta)a_2^2] = 2\Upsilon_1 P_1(h_1 - k_1) + 2\Upsilon_0 P_1(h_2 - k_2) + \Upsilon_0(P_2 - P_1)(h_1^2 - k_1^2). (3.26)$$

Combining the relation $(3.26)$ with the estimate obtained in $(3.25)$, we arrive at:

$$|a_3| \leq \frac{|\Upsilon_0|P_2}{(1+6\varepsilon+3\varepsilon\delta+\frac{\varepsilon(\varepsilon-1)}{2}(2+\delta)^2)} + \frac{P_1(|\Upsilon_0|+|\Upsilon_1|)}{(2+6\varepsilon+4\varepsilon\delta)}.$$

Furthermore, another estimate for $|a_3|$ is obtained as follows:

$$|a_3| \leq \frac{P_1(|\Upsilon_0|+|\Upsilon_1|)}{(2+6\varepsilon+4\varepsilon\delta)} + \frac{|\Upsilon_0|^2 P_1^2}{(1+\varepsilon(2+\delta))^2}.$$

The proof is complete. ■
Setting $\delta = 0$ in Theorem $(3.1)$, leads to the result below.
**Corollary (3.1):** Let $f \in H$ belong $\ell_{q,\Sigma}(0, \varepsilon, \varphi)$. Then

$$|a_2| \leq min\{\frac{|\Upsilon_0|P_1}{(1+2\varepsilon)}, \sqrt{\frac{|\Upsilon_0|P_2}{1+6\varepsilon+2\varepsilon(\varepsilon-1)}}\},$$

and

$$|a_3| \leq min\{\frac{|\Upsilon_0|P_2}{(1+6\varepsilon+2\varepsilon(\varepsilon-1))} + \frac{P_1(|\Upsilon_0|+|\Upsilon_1|)}{(2+6\varepsilon)}, \frac{P_1(|\Upsilon_0|+|\Upsilon_1|)}{(2+6\varepsilon)} + \frac{|\Upsilon_0|^2 P_1^2}{(1+2\varepsilon)^2}\}.$$

Putting $\varepsilon = 1$ and $\delta = 0$ in Theorem $(3.1)$, leads to the result below.
**Corollary (3.2):** Let $f \in H$ belong to $\ell_{q,\Sigma}(\delta, \varepsilon, \varphi)$. Then

$$|a_2| \leq min\{\frac{1}{3}|\Upsilon_0|P_1, \sqrt{\frac{1}{7}|\Upsilon_0|P_2}\},$$

and

$$|a_3| \leq min\{\frac{1}{7}|\Upsilon_0|P_2 + \frac{1}{8}P_1(|\Upsilon_0| + |\Upsilon_1|), \frac{1}{8}P_1(|\Upsilon_0| + |\Upsilon_1|) + \frac{1}{9}|\Upsilon_0|^2 P_1^2\}.$$

**Conclusion**

In the presen sttudy, we introduced two distinct subclasses within the family of analytic bi-univalent functions, $\mathcal{M}_{q,\Sigma}^{\beta}(\varsigma, \mu, \varphi)$ and $\ell_{q,\Sigma}(\delta, \varepsilon, \varphi)$, based on quasi-subordination. For each class, we obtained bounds for the coefficients $|a_2|$ and $|a_3|$by using certain properties of functions with positive real parts and comparing the coefficients of the relevant series expansions. We also considered some special cases by assigning particular values to the parameters involved in the definitions. The estimates obtained are not necessarily sharp. Further work may focus on finding sharp bounds for these coefficients, deriving estimates for higher-order coefficients, together with investigating Fekete–Szegö inequalities for the two classes.

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